63. $\int \frac{1}{x}dx = \frac{1}{1+(x-1)} = \int 1-(x-1)+(x-1)^2-(x-1)^3 \dots dx$

$a=1 \quad r=-(x-1) \qquad |-(x-1)|<1 \qquad -1<x-1<1$

$0<x<2$

$\ln x = x - \frac{(x-1)^2}{2} + \frac{(x-1)^3}{3} - \frac{(x-1)^4}{4} \dots + c$

$0=1+c \quad c=-1$

$\ln x = (x-1) - \frac{(x-1)^2}{2} + \frac{(x-1)^3}{3} \dots$

$= \sum_{n=1}^{\infty} \frac{(x-1)^n}{n}(-1)^{n+1}$

Feb 1-10:05 AM

33. $\int_0^x f(t)\,dt = t - \frac{(t-3)^2}{4} + \frac{(t-3)^3}{12} \dots \Big|_0^x$

$\boxed{\left(x - \frac{(x-3)^2}{4} + \frac{(x-3)^3}{12} \dots\right) - \left(\frac{(-3)^2}{4} + \frac{(-3)^3}{12} + \dots\right)}$

$f(x) = 1 - \frac{x-3}{2} + \frac{(x-3)^2}{4} \dots$

$a=1 \quad r=-\frac{(x-3)}{2}$

23. $\int \sum \left(-\frac{1}{2}\right)^n (x-3)^n dx = \sum \left(-\frac{1}{2}\right)^n \frac{(x-3)^{n+1}}{n+1} + c$

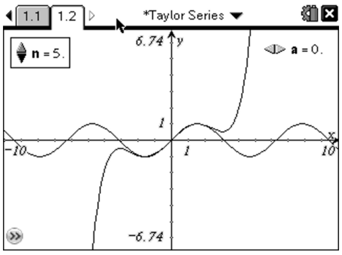
$\left|\frac{x-3}{2}\right| < 1$

$\int \frac{1}{2}x^n\,dx$

Feb 1-10:15 AM

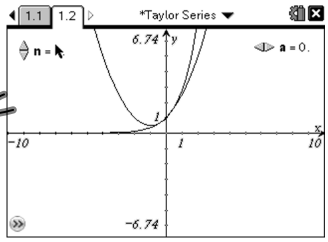
9.2a Taylor Series

How can we find a polynomial that looks like another function?



Jan 28-10:59 AM

Ex. Find a parabola that fits y=e^x for x near 0



$f(x)=e^x$ $\quad p(x)=ax^2+bx+c$

$f(0)=p(0)$ $\quad p'(x)=2ax+b$

$\boxed{1=e^0=c}$ $\quad p''(x)=2a$

$f'(0)=p'(0)$

$\boxed{1=e^0=b}$ $\quad p(x)=\frac{1}{2}x^2+x+1$

$f''(0)=p''(0)$

$1=e^0=2a$

$\boxed{a=\frac{1}{2}}$

Jan 28-11:04 AM

Find an nth degree polynomial that fits y=e^x for x near 0

$p(x) = c_0 + c_1 x + c_2 x^2 + c_3 x^3 \ldots + c_n x^n \ldots$

$f(x) = e^x \quad f(0) = 1 = p(0) = c_0$

$f'(x) = e^x \quad f'(0) = 1 = p'(0) = c_1$

$f''(x) = e^x \quad f''(0) = 1 = p''(0) = 2c_2 \quad c_2 = \frac{1}{2}$

$f^3(x) = e^x \quad f'''(0) = 1 \quad p'''(0) = 3 \cdot 2 c_3 \quad c_3 = \frac{1}{3 \cdot 2}$

$c_4 = \frac{1}{4 \cdot 3 \cdot 2} = \frac{1}{4!}$

$c_5 = \frac{1}{5!}$

$p'(x) = c_1 + 2c_2 x + 3c_3 x^2 \ldots$

$p''(x) = 2c_2 + 3 \cdot 2 c_3 x + \ldots$

$p'''(x) = 3 \cdot 2 c_3 \ldots$

$e^x = 1 + x + \frac{x^2}{2} + \frac{x^3}{3!} + \frac{x^4}{4!} + \frac{x^5}{5!} \ldots = \sum_{n=0}^{\infty} \frac{x^n}{n!}$

$e^{x^2} = 1 + x^2 + \frac{(x^2)^2}{2} + \frac{(x^2)^3}{3!} \ldots$

Jan 28-11:02 AM

Construct a 4th degree polynomial that matches y=ln(1+x) at x=0

Jan 28-11:35 AM

Maclaurin Series for f(x)

$$f(x) = \overbrace{f(0) + f'(0)x + \frac{f''(0)}{2}x^2 + \frac{f'''(0)}{3!}x^3 + \frac{f^{4}(0)}{4!}x^4}^{P(x)} + \dots + \frac{f^{n}(0)}{n!}x^n \dots$$

Jan 28-11:03 AM

Find the Maclaurin series for f(x)=sin(x). How many terms are required to approximate sin(7) accurate to the third decimal place?

$f(x) = \sin x$ | $f(0) = 0$

$f'(x) = \cos x$ | $f'(0) = 1$

$f''(x) = -\sin x$ | $f''(0) = 0$

$f^3(x) = -\cos x$ | $f'''(0) = -1$

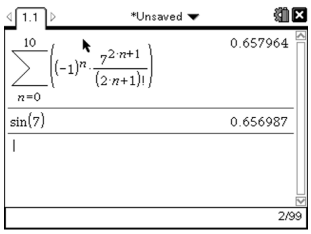
$f^4(x) = \sin x$ | $f^4(0) = 0$

$f^5(x) = \cos x$ | $f^5(0) = 1$

$f^6(x) = -\sin x$ | $f^6(0) = 0$

$f^7(x) = -\cos x$ | $f^7(0) = -1$

$f^8(x) = \sin x$



$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \frac{x^9}{9!} - \frac{x^{11}}{11!}$$

$$\sin(2x) = (2x) - \frac{(2x)^3}{3!} + \frac{(2x)^5}{5!} \dots$$

Jan 28-11:37 AM